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FROM MISCONCEPTION TO CONCEPTUAL UNDERSTANDING: HYPOTHETICAL LEARNING TRAJECTORY DESIGN IN SIMILARITY LEARNING

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ABSTRACT

Similarity is a fundamental topic in geometry, yet many students continue to experience difficulties in understanding proportional relationships, identifying corresponding elements, and constructing geometric reasoning. This study aimed to synthesize empirical evidence on students' difficulties in learning geometric similarity and to develop a Hypothetical Learning Trajectory (HLT) to support instruction. A Systematic Literature Review (SLR) was conducted following the PRISMA 2020 guidelines. Data were collected from six databases: Scopus, ScienceDirect, SpringerLink, Taylor & Francis Online, ERIC, and Google Scholar. From 142 identified records published between 2015 and 2025, 15 empirical studies met the inclusion criteria and were selected for analysis. Data were analyzed using thematic synthesis through open coding, axial coding, and selective coding. The findings revealed six major categories of students' difficulties: conceptual errors, transformation errors, procedural errors, visual–spatial difficulties, metacognitive misconceptions, and reasoning-and-proof difficulties. These difficulties form a developmental pattern, progressing from visual and conceptual misunderstandings to higher-order reasoning challenges. Based on the synthesis, a five-phase Hypothetical Learning Trajectory consisting of visual recognition, correspondence identification, proportional reasoning, contextual application, and deductive justification was developed. The proposed HLT provides an evidence-based framework for improving geometry instruction and supporting students' conceptual understanding of similarity.

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INTRODUCTION

Geometry serves as a cornerstone of mathematics education, forming the basis for the development of advanced spatial, deductive, and representational reasoning skills (Clements & Battista, 1989; Jones & Tzekaki, 2016). Among its fundamental domains, the topic of similarity has persistently presented conceptual difficulties for students. A growing body of international and national research indicates that learners often struggle to grasp core ideas of similarity such as identifying corresponding sides, applying proportional relationships, and distinguishing between similarity and congruence (Osibodu & Sunghwan Byun, 2021; Solihah & Muhtadi, 2025; Sulastri et al., 2017). Findings from PISA 2022 further reveal that students' spatial reasoning and proportional thinking in geometry remain relatively weak across countries, particularly in Indonesia, where many rely heavily on visual perception rather than conceptual understanding (OECD, 2024). This evidence underscores a persistent gap between students' cognitive structures and the conceptual demands embedded within the geometry curriculum.

From a cognitive perspective, students' difficulties in understanding similarity can be interpreted through the integration of five major theoretical frameworks in mathematics education those Hiele (1986) and Piaget (970), through successive levels of visualization, analysis, informal deduction, formal deduction, and rigor. However, many secondary students remain at the visualization analysis stages, while instructional approaches often expect reasoning at the formal deductive level (Gutiérrez, A., & Jaime, 2021; Usiskin, 1982). This misalignment results in an intuitive rather than conceptual understanding of similarity (Duval, 1998; Mariotti & Fischbein, 1997). From Piaget (1970) such challenges stem from the fact (For et al., 2006)(For et al., 2006)that numerous students still operate at the concrete operational stage and have not yet attained formal operational reasoning, which is essential for grasping abstract geometric relationships (Battista, 2007; Piaget, 1958).

Vygotsky (1978) framework of the Zone of Proximal Development (ZPD) underscores that the transition from intuitive to formal reasoning requires deliberate social mediation, typically facilitated through teacher scaffolding and peer collaboration. Empirical evidence suggests that peer scaffolding and dialogic reasoning practices can effectively accelerate students' movement from visual to analytical levels of geometric thinking (Goos,, Galbraith, & Renshaw, 2002).

In relation to mathematical problem solving, Polya et al (1945) classic four-step model understanding the problem, devising a plan, carrying out the plan, and reviewing the process together with Schoenfeld (1985) metacognitive extensions, highlight the critical role of reflective thinking. Persistent difficulties at the initial stage of "understanding the problem" often trigger misconceptions, particularly in contextual geometry tasks (Fadilah & Bernard, 2021; Stylianides et al., 2024). Complementing this, Newman (1977) provides a diagnostic framework to trace students' errors from comprehension to response construction. This analytic approach has proven particularly effective in identifying both transformational and comprehension errors in learning similarity concepts (Nafiin, 2020; Nurmala, & Jupri, 2023).

When synthesized, these five theoretical perspectives establish a comprehensive cognitive didactical foundation for interpreting student errors not merely as failures but as indicators of ongoing cognitive development. Such an integrative stance aligns with Tall (2004) conceptualization of the "three worlds of mathematics" embodied, symbolic, and formal which emphasizes that learners transition progressively from visual representations to symbolic reasoning and ultimately to deductive abstraction, guided by conceptual reflection (Dreyfus, 2015; Tall, 2011). Within this framework, conceptual errors are viewed as natural markers of students' operation within a mathematical world distinct from that expected by formal instruction, rather than as deficiencies to be corrected.

To bridge the divide between learning theory and classroom practice, the framework of Hypothetical Learning Trajectory (HLT) initially conceptualized by (Simon, 1995) and later refined by (Gravemeijer, & Cobb, 2006) offers a systematic model for designing conceptually grounded instruction. HLT delineates the anticipated developmental pathway of students' understanding through three essential components: (1) the learning goals, (2) the hypothetical progression of learners' thinking, and (3) the instructional activities intended to facilitate that progression (Bakker & Smit, 2018; Irawan et al., 2025). This design-oriented approach has been widely applied in mathematics education research to construct learning trajectories for topics such as functions (Nogueira et al., 2021), fractions (Stark Guss et al., 2022), and geometric transformations (Bakker & Smit, 2018; Gravemeijer & Cobb, 2006). Doorman, demphasized the inclusion of reflective and diagnostic dimensions within HLT frameworks to strengthen the connection between student errors and the dynamics of learning progression (Alonzo & von Aufschnaiter, 2018; Hiebert *et al.,* 2018; Mosher, 2022).

Nevertheless, the application of HLT to the topic of geometric similarity remains limited, particularly within the Indonesian mathematics education context, which is grounded in a constructivist curriculum framework. Most national studies have concentrated on identifying and classifying students' misconceptions without explicitly linking these findings to theoretically informed and prospective learning trajectories (Didi et al., 2019; Firdaus & Mutaqin, 2019). This highlights a persistent research gap between two major paradigms in mathematics education research : (a) diagnostic studies, which focus on analyzing what students misunderstand, and (b) didactical design research, which seeks to explore how instructional interventions can be structured to overcome such misunderstandings (Artigue, 2009; Gravemeijer & Doorman, 1999). Addressing this divide is crucial for developing pedagogical models that align diagnostic insight with design-based instructional innovation.

The present research concentrates on the initial phase, prospective analysis, aimed at constructing a hypothetical learning trajectory (HLT) for the concept of similarity grounded in diagnostic insights into students' errors. Through this methodological lens, a conceptual prototype termed the Diagnostic Reflective HLT Model was developed, positioning student errors not as mere procedural mistakes but as meaningful indicators of cognitive development and conceptual growth.

The Diagnostic Reflective HLT Model integrates three core principles that collectively redefine the role of student error within the learning process. The diagnostic principle focuses on identifying students' cognitive structures through the analysis of recurring error patterns; the reflective principle emphasizes transforming these errors into opportunities for conceptual reflection and self-regulation; and the didactical principle guides the design of adaptive learning activities aligned with students' current developmental stages.

This framework resonates with the learning from errors paradigm (Borasi, 1994) and the growing movement toward reflective mathematical teaching within global mathematics education research (Sinclair & Bruce, 2015; Stylianides et al., 2024). Consequently, the study offers a twofold contribution: theoretically, by extending the applicability of HLT to geometric contexts, and practically, by proposing a pedagogical model that situates error analysis as both diagnostic and transformative in nature.

In summary, this study moves beyond merely identifying the sources of students' errors to exploring how such errors can be harnessed as catalysts for deeper conceptual understanding. By reconceptualizing errors as productive learning indicators, the proposed Diagnostic Reflective HLT Model bridges the divide between diagnostic research and didactical design research.

Grounded in the integrative synthesis of five foundational theories Van Hiele's geometric thought levels, Vygotsky's sociocultural scaffolding, Polya's problem-solving heuristics, Piaget's developmental stages, and Newman's error analysis the model aligns with the contemporary trajectory of mathematics education research that emphasizes reflective and transformative learning (Ginting et al., 2025; Osta, 2014; Silmi & L, 2022).

The theoretical implications lie in re-framing student misconceptions as developmental transitions rather than deficits, while the practical contribution emerges through a didactical framework that supports adaptive, reflection-driven instruction within geometry education. This approach underscores the potential of diagnostic reflective design as a means to foster metacognitive growth and promote sustainable conceptual change among learners.

METHOD

This study employed a Systematic Literature Review (SLR) approach to synthesize empirical evidence regarding students' misconceptions, learning obstacles, reasoning difficulties, and instructional responses in learning geometric similarity. The primary objective of this review was to construct a conceptual Hypothetical Learning Trajectory (HLT) grounded in empirical findings and supported by established theories in mathematics education.

The SLR approach was selected because it enables a comprehensive, transparent, and replicable synthesis of existing knowledge while identifying research trends, gaps, and implications for educational practice (Snyder, 2019). To ensure methodological rigor, the review process followed the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA 2020) guidelines proposed by (Page et al, 2021).

The overall research procedure consisted of six interconnected stages: (1) formulation of research questions, (2) literature search, (3) screening and eligibility assessment, (4) data extraction, (5) coding and thematic synthesis, and (6) construction of the conceptual Hypothetical Learning Trajectory (HLT). To facilitate readers' understanding of the methodological workflow and to enhance research transparency, the complete research procedure is illustrated in figure 1.

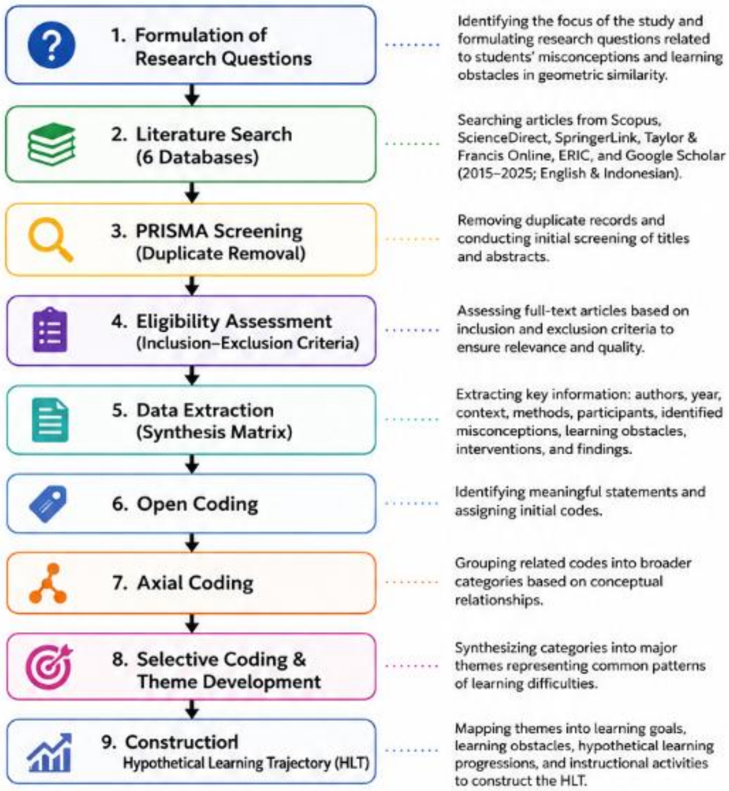

Figure 1. Research Procedure Flowchart

The literature search was conducted across six internationally recognized academic databases, namely Scopus, ScienceDirect, SpringerLink, Taylor & Francis Online, ERIC, and Google Scholar. These databases were selected due to their extensive coverage of educational research, indexing credibility, and accessibility to full-text articles.

To capture contemporary developments in mathematics education research, the search was limited to studies published between 2015 and 2025. Only articles published in English and Indonesian were included to ensure both international relevance and contextual alignment with mathematics education research in Indonesia.

The search period was limited to publications between 2015 and 2025 to capture recent developments in research on geometric misconceptions, learning of similarity, and the application of HLT. Only studies published in English and Indonesian were included to ensure both international relevance and contextual alignment with mathematics education in Indonesia.

Search strings were formulated using combinations of Boolean operators and conceptual synonyms related to geometric similarity, misconceptions, learning obstacles, and learning trajectories. The search included terms such as "geometric similarity," "student misconceptions," "learning obstacles," "geometry education," and "hypothetical learning trajectory." All retrieved records were imported into Rayyan.ai (Ouzzani, 2016). Bibliographic data were subsequently exported to Mendeley and Zotero for reference management. All search outputs and screening decisions were documented in a digital audit trail to support research transparency and replicability.

The inclusion and exclusion criteria were defined following the guidelines proposed (Riyan et al., 2023). Studies were included if they employed qualitative, quantitative, or mixed-methods approaches; investigated geometric similarity or closely related concepts; examined students' misconceptions, errors, learning obstacles, or reasoning difficulties; involved participants ranging from elementary school students to pre-service mathematics teachers; were published in peer-reviewed journals indexed in Scopus or equivalent databases; and were available in full-text format. Studies were excluded if they were conceptual or theoretical papers, conference abstracts without complete manuscripts, studies unrelated to geometry or similarity concepts, duplicate publications, or articles lacking sufficient methodological information.

The study selection process followed the four stages recommended by the PRISMA 2020 framework (Page et al., 2021) During the identification stage, 142 records were retrieved from the selected databases. Following duplicate removal, 78 studies remained for title and abstract screening. Subsequently, full-text eligibility assessments were conducted based on relevance, methodological quality, and alignment with the inclusion criteria, resulting in 38 potentially eligible studies. After a detailed evaluation, 15 studies were retained for the final synthesis.

The screening and eligibility assessments were independently conducted by two reviewers. Inter-rater reliability was calculated using Cohen's Kappa (κ), yielding a coefficient of $\kappa = 0.86$, which indicates a high level of agreement (Saldaña, et al., 2014). The complete study selection process is illustrated in Table 1.

Table 1. PRISMA-Based Article Selection and Review Process

Stage	Review Procedure	Output
Identification	Retrieval of publications related to geometric similarity, misconceptions, learning obstacles, and HLT from six international databases (Scopus, ScienceDirect, SpringerLink, Taylor & Francis Online, ERIC, and Google Scholar)	142
Deduplication	Removal of duplicate records and non-relevant database entries	64
Screening	Evaluation of titles and abstracts according to predefined inclusion criteria	78
Exclusion	Removal of studies not addressing the conceptual focus of the review	40
Eligibility	Full-text articles assessed for methodological quality and thematic relevance	38
Exclusion	Removal of studies failing to meet quality and relevance requirements	23
Inclusion	Studies included in thematic synthesis and HLT construction	15

Data extraction was conducted using a structured extraction protocol developed by the researchers. Information extracted from each study included authorship, publication year, educational context, research objectives, participant characteristics, methodological approach, identified misconceptions and learning obstacles, instructional interventions, and major findings. The extracted information was organized into a synthesis matrix to facilitate systematic comparison and analysis across studies.

To ensure analytical transparency and traceability, thematic synthesis was conducted using NVivo 14 software. The coding process followed the thematic analysis framework proposed by Braun and Clarke (2021), consisting of three iterative stages: open coding, axial coding, and selective coding.

During the open coding stage, meaningful statements related to students' misconceptions, reasoning difficulties, learning obstacles, and instructional responses were identified and assigned initial descriptive codes. Examples of these initial codes included proportional reasoning error, corresponding-side confusion, visual dominance, scale-factor misunderstanding, symbol interpretation error, and transformation misconception.

In the axial coding stage, conceptually related codes were grouped into broader categories by identifying recurring relationships and patterns across studies. These categories included proportional reasoning difficulties, visual-spatial misconceptions, representational misunderstandings, procedural errors, and instructional support needs.

Subsequently, selective coding was conducted to synthesize these categories into overarching themes representing common patterns of learning difficulties in geometric similarity. Four major themes emerged from the analysis: (1) difficulties in understanding proportional relationships, (2) visual-spatial reasoning misconceptions, (3) symbolic and representational misunderstandings, and (4) difficulties in applying similarity concepts within problem-solving contexts.

To enhance analytical transparency and ensure that the coding process can be traced by readers, examples of the transformation from raw findings to themes are presented in Table 2.

Table 2 Example of Coding and Theme Extraction Process

Raw Findings	Initial Code	Category	Theme
Students confused corresponding sides when solving similarity problems	Corresponding side confusion	Proportional reasoning difficulties	Difficulties in understanding proportional relationships
Students interpreted the symbol "//" as indicating equal length rather than parallel lines	Symbol interpretation error	Representational misunderstanding	Symbolic and representational misunderstandings
Students relied solely on visual appearance when comparing geometric figures	Visual dominance	Visual-spatial misconception	Visual-spatial reasoning misconceptions
Students applied additive reasoning instead of proportional reasoning	Additive strategy error	Proportional reasoning difficulties	Difficulties in understanding proportional relationships
Students failed to transfer similarity concepts into contextual problems	Contextual transfer difficulty	Problem-solving obstacle	Difficulties in applying similarity concepts

The coding process was conducted iteratively until thematic saturation was achieved. Throughout the analysis, coding decisions were continuously compared across studies to ensure consistency and coherence. Any discrepancies between reviewers were discussed until consensus was reached. The resulting themes were subsequently mapped onto learning goals, anticipated learning obstacles, hypothetical progressions of students' thinking, and instructional activities to construct the conceptual Hypothetical Learning Trajectory (HLT).

The construction of the HLT was informed by the integration of the matic synthesis findings with major theoretical perspectives in mathematics education, including Van Hiele's theory of geometric thinking, Piaget's cognitive development theory, Vygotsky's sociocultural theory, Polya's problem-solving framework, and Newman's error analysis. The identified themes were systematically linked to anticipated conceptual development pathways, enabling the formulation of an empirically grounded and theoretically informed HLT for learning geometric similarity.

The trustworthiness of the findings was strengthened through researcher triangulation, theory triangulation, and methodological triangulation Patton (2015) Researcher triangulation was achieved through independent coding and verification by two reviewers. Theory triangulation involved interpreting findings through multiple theoretical perspectives, while methodological triangulation combined thematic synthesis and content analysis. In addition, two mathematics education experts reviewed the coding framework, thematic interpretations, and HLT construction process to ensure conceptual coherence and analytical credibility.

Because this study relied exclusively on secondary data obtained from published literature and did not involve human participants, formal ethical approval was not required. To further support transparency and reproducibility, all stages of the review process, including search strategies, inclusion–exclusion decisions, coding outputs, thematic synthesis records, and PRISMA documentation, were systematically archived. This practice aligns with contemporary open science principles that emphasize transparency, reproducibility, and accountability in educational research (Jabeen, 2025).

RESULT AND DISCUSSION

RESULT

1. Overview

This Systematic Literature Review (SLR) synthesized findings from fifteen empirical studies published between 2015 and 2025 concerning the teaching and learning of geometric similarity. The reviewed studies involved participants ranging from elementary school students to pre-service mathematics teachers and employed qualitative, quantitative, and mixed-method research designs.

The synthesis revealed that students' difficulties in learning similarity are multidimensional and interconnected rather than isolated phenomena. Through a systematic coding process, recurring patterns of errors were identified and subsequently grouped into broader themes representing major learning obstacles in understanding geometric similarity.

The analytical process consisted of three stages: open coding, axial coding, and selective coding. Open coding was used to identify recurrent findings reported across the selected studies. Similar codes were then grouped through axial coding into categories of learning difficulties. Finally, selective coding was employed to synthesize these categories into overarching themes that explain common patterns of students' errors in learning similarity.

2. Coding and Theme Extraction Process

To ensure analytical transparency and traceability, all empirical findings extracted from the reviewed studies were analyzed using NVivo-assisted thematic analysis.

Table 3. Coding and Theme Extraction Process

Open Codes	Axial Codes	Selective Themes
Incorrect identification of corresponding sides	Incomplete conceptual understanding	Conceptual Errors
Judging similarity based on visual appearance	Incomplete conceptual understanding	Conceptual Errors
Incorrect proportional models	Mathematical representation difficulties	Transformation Errors
Misapplication of ratios	Mathematical representation difficulties	Transformation Errors
Computational mistakes	Procedural difficulties	Procedural Errors
Incomplete solution procedures	Procedural difficulties	Procedural Errors
Distorted geometric drawings	Visual–spatial obstacles	Visual–Spatial Difficulties
Misinterpretation of geometric symbols	Visual–spatial obstacles	Visual–Spatial Difficulties
High confidence in incorrect answers	Weak self-monitoring	Metacognitive Misconceptions
Weak logical justification	Deductive reasoning difficulties	Reasoning and Proof Difficulties

The coding process generated five dominant themes: conceptual errors, transformation errors, procedural errors, visual–spatial difficulties, and metacognitive misconceptions. These themes consistently appeared across multiple studies and represent recurring learning obstacles in students' understanding of geometric similarity.

3. Patterns of Students' Errors in Learning Geometric Similarity

3.1 Conceptual Errors

Conceptual errors emerged as the most dominant type of difficulty across the reviewed studies. Many students demonstrate an inadequate understanding of the meaning of similarity and the necessary conditions under which two geometric figures can be considered similar.

Nafiin (2020) reported that 57.6% of students were unable to correctly identify the defining properties of similarity. For example, when asked to determine whether triangles ΔABC and ΔDEF are similar, several students wrote:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

even though side FD does not correspond to side CA. Researchers noted that students tended to match sides based on perceived positional similarity rather than on angle correspondence. Similar findings were (Solihah & Muhtadi 2025). where students assumed that triangles that "look alike" must be similar, even when proportional relationships between corresponding sides were not satisfied.

A representative student interview excerpt stated: "The shapes look the same, so they must be similar." (Solihah & Muhtadi, 2025 p.7). These findings indicate a strong reliance on visual perception without conceptual grounding, suggesting that students' reasoning is driven more by appearance than by mathematical properties.

3.2 Transformational Errors

Procedural and transformational errors commonly occurred when students failed to translate problem statements into appropriate mathematical representations or incorrectly applied proportional relationships.

Nafiin (2020) identified frequent occurrences of *reading errors*, *transformation errors*, and *process skill errors*. For instance, in a problem stating:

"Triangle ABC is similar to triangle DEF. If AB = 4 cm, AC = 6 cm, and DE = 8 cm, determine DF,"

some students wrote:

$$\frac{AB}{DE} = \frac{AC}{EF}$$

and subsequently calculated

$$EF = \frac{8 \times 6}{4} = 12,$$

despite EF being neither given nor required. This error was categorized as a transformation error, indicating students' inability to correctly identify corresponding relationships among sides.

Similarly, Laila et al (2025) reported a transformation error rate of 41.25% among preservice mathematics teachers. A recurring misconception involved statements such as: "Because all sides are equal in length, the triangles are similar," which actually reflects the concept of congruence rather than similarity. These findings suggest weak mathematical modelling skills, where students can perform numerical calculations but fail to construct logically consistent geometric representations.

3.3 Visual Spatial Difficulties

Visual spatial difficulties were identified as significant learning obstacles in several studies (Firdaus & Mutaqin, 2019), reported four types of visual learning obstacles prior to didactical intervention. For example, when students were asked to draw two similar triangles with a scale factor of 1:2, some produced figures with random and non-proportional side lengths. When questioned, a student responded: "I thought it was enough if they looked similar; the exact proportions were not necessary." (Firdaus & Mutaqin, 2019).

Further evidence showed that many students misinterpreted the geometric symbol "//" as indicating equal side lengths rather than parallelism: *"I thought // meant the same length, not parallel"* (Murio, 2022). This misconception reflects students' difficulties in coordinating visual representations with formal geometric conventions. The findings suggest that weaknesses in symbolic interpretation and visual–spatial reasoning may hinder students' understanding of proportional relationships in similarity concepts.

3.4 Procedural Errors

Several studies revealed that students struggle with contextual geometry problems involving similarity. Rohmah & Rosyidi (2022) found that students failed at all four stages of Polya's problem-solving framework.

For example, in the problem: "A pole 6 meters tall casts a shadow 4 meters long. If a tree casts a shadow 10 meters long, how tall is the tree?" some students wrote:

$$\frac{6}{4} = \frac{4}{10} \Rightarrow x = 2.4$$

and concluded that the tree's height was 2.4 meters instead of the correct value of 15 meters. The error resulted from incorrect pairing of corresponding quantities between height and shadow length.

Fadilah & Bernard (2021) and Sulastri et al (2017) reported that 51.4% of students failed to understand similar contextual problems. Classroom observations indicated that students often copied numerical values directly into formulas without considering the physical meaning of the objects involved. This reinforces evidence of weak translation skills between real-world situations and mathematical representations.

3.5 Errors in Proof and Deductive Reasoning

At the secondary and tertiary levels, multiple studies identified substantial difficulties in constructing formal geometric proofs. Daniyati et al (2023) reported that only 50.9% of students were able to produce logically coherent proofs. A common incorrect justification was: "Because the sides are in proportion, the triangles are automatically similar," without reference to angle equivalence or established similarity theorems. (Al-baqie & Budiarto, 2022) similarly found that students who failed in proving similarity were unable to articulate logical reasoning at each step. One student stated: "I remembered a similar example before, so I just wrote that they are the same." These findings indicate weak *logical chaining* and *deductive reasoning*, which correspond to higher Van Hiele levels (levels 4–5) of geometric thinking.

3.6 Metacognitive Misconceptions

Metacognitive misconceptions were also prevalent. Hamzah et al (2019) using the *Certainty of Response Index* (CRI), found that 53.34% of students exhibited conceptual misconceptions. For example, students correctly stated that "two figures are similar if their corresponding sides are proportional," but justified their answer by stating:

"Because their areas must also be the same," with a confidence level of 5 out of 5. This represents a *false-positive misconception*, where students are highly confident in incorrect reasoning. Such findings highlight deficiencies in metacognitive regulation, particularly in students' ability to monitor and evaluate their own understanding.

4. Cross-Study Thematic Synthesis

To identify recurring patterns across studies, thematic aggregation was conducted based on the number of articles reporting each theme. It should be noted that the frequencies presented in Table 4 refer to the number of studies reporting a particular theme rather than percentages of students

Table 4. Thematic Synthesis of Students' Difficulties in Learning Geometric Similarity

Error Type	Representative Evidence	Number of Studies (n = 15)	Interpretation
Conceptual Errors	Incorrect side correspondence; judging similarity based on visual appearance	12	Most frequently reported difficulty
Transformation Errors	Incorrect construction of proportional relationships	9	Indicates weaknesses in mathematical modelling

Error Type	Representative Evidence	Number of Studies (n = 15)	Interpretation
Procedural Errors	Computational mistakes and incomplete solution procedures	8	Frequently follows conceptual misunderstanding
Visual–spatial Difficulties	Distorted drawings and misinterpretation of symbols	6	Related to spatial reasoning limitations
Reasoning and Proof Difficulties	Inability to construct logical arguments and formal geometric proofs	5	Indicates limited deductive reasoning and difficulties at higher Van Hiele levels
Metacognitive Misconceptions	High confidence in incorrect answers	4	Indicates limited self-monitoring ability

Table 4 presents the thematic synthesis derived from the coding and theme extraction process. The frequencies shown in the table represent the number of reviewed studies reporting each category of difficulty rather than the percentage of students experiencing the difficulty. Conceptual errors emerged as the most frequently reported theme, appearing in 12 of the 15 reviewed studies. This finding suggests that misunderstanding the fundamental properties of similarity constitutes the primary obstacle underlying many subsequent errors, including transformation, procedural, and reasoning difficulties.

5. Synthesis of Students' Errors Based on Newman Error Analysis Framework

In addition to the thematic synthesis, this study also conducted a synthesis based on the Newman Error Analysis (NEA) framework, which was adopted in several of the reviewed studies. Newman's framework categorizes students' errors into five stages: reading, comprehension, transformation, process skills, and encoding. This synthesis was undertaken to identify the stages at which students most frequently encounter difficulties when solving geometric similarity problems.

It is important to note that the frequencies presented in Table 5 refer to the number of reviewed studies reporting a particular error category, rather than the percentage of students experiencing that error. This approach was employed to provide a systematic cross-study comparison and to identify recurring patterns of difficulties reported in the literature

Table 5. Synthesis of Students' Errors Based on Newman's Error Analysis Framework

Newman Category	Number of Studies	Representative Evidence
Reading Errors	3	Misinterpretation of mathematical symbols and terminology
Comprehension Errors	8	Failure to identify known and unknown information
Transformation Errors	9	Incorrect construction of proportional models and correspondence relationships
Process Skill Errors	7	Procedural and computational mistakes during problem solving
Encoding Errors	5	Final answers inconsistent with the problem context

Table 5 indicates that transformation errors were the most frequently reported category among studies employing Newman's framework, appearing in nine reviewed articles. This finding suggests that many students experience difficulties in translating verbal or visual information into appropriate mathematical representations. Comprehension errors were also frequently identified, indicating that students often struggle to recognize relevant information and understand problem requirements before initiating solution procedures.

In contrast, reading errors and encoding errors appeared less frequently across the reviewed studies. Nevertheless, these difficulties remain important because they can influence subsequent stages of problem solving. Overall, the findings suggest that the primary challenges in learning geometric similarity are not merely computational in nature, but are more closely related to students' abilities to comprehend, represent, and mathematically model geometric situations.

6. Developmental Pattern of Students' Errors

One of the most significant findings of this review is that students' errors do not occur independently but form a developmental sequence.

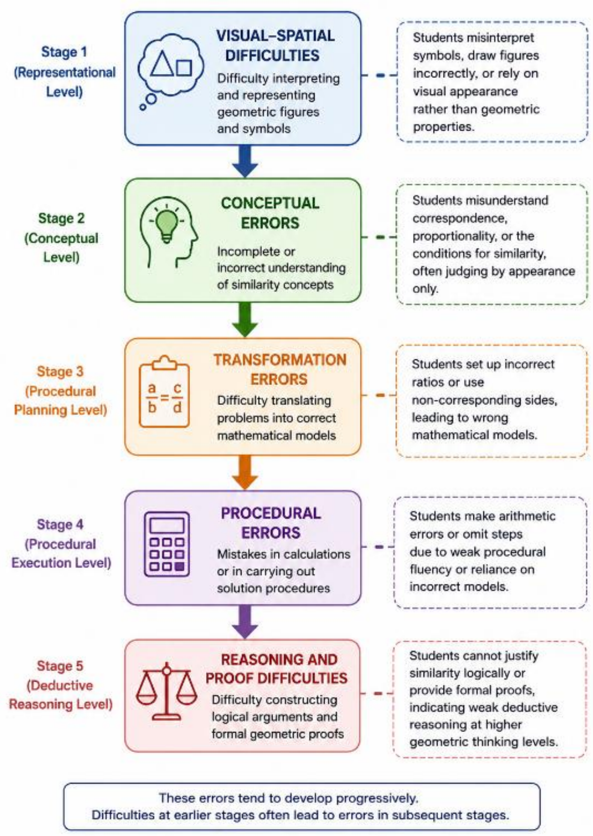

Figure 2. Developmental Pattern of Students' Errors in Learning Geometric Similarity

This pattern suggests that difficulties in interpreting visual representations frequently trigger conceptual misunderstandings. These misunderstandings subsequently lead to transformation errors when students attempt to construct mathematical models. Incorrect models then contribute to procedural mistakes and eventually hinder students' ability to formulate formal deductive justifications. Thus, students' errors should be viewed as interconnected manifestations of developing mathematical thinking rather than isolated .

7. Construction of the Hypothetical Learning Trajectory (HLT)

Based on the synthesis of learning obstacles identified across the reviewed studies, this study proposes a Hypothetical Learning Trajectory (HLT) for learning geometric similarity.

The HLT was constructed by aligning each identified learning obstacle with instructional responses intended to facilitate conceptual progression.

Table 6. Proposed Hypothetical Learning Trajectory for Geometric Similarity

HLT Phase	Learning Goal	Anticipated Difficulties	Learning Activities
Visual Recognition	Distinguish visual resemblance from similarity	Visual misconceptions	Comparing similar and non-similar figures
Correspondence Identification	Identify corresponding sides and angles	Conceptual errors	Matching corresponding elements
Proportional Reasoning	Understand scale factors and proportional relationships	Transformation errors	Ratio exploration activities
Contextual Application	Apply similarity in real-life situations	Procedural difficulties	Context-based problem solving
Deductive Justification	Construct formal explanations and proofs	Reasoning and proof difficulties	Mathematical argumentation and proof tasks

The proposed HLT suggests that students' understanding of similarity should develop progressively from visual recognition to formal deductive reasoning through a sequence of increasingly sophisticated learning experiences.

DISCUSSION

1. General Interpretation of Findings

The findings of this systematic literature review reveal that students' difficulties in learning geometric similarity are multidimensional, involving conceptual understanding, mathematical representation, procedural execution, metacognitive regulation, and deductive reasoning. The synthesis of fifteen empirical studies consistently identified six dominant categories of difficulties: conceptual errors, transformation errors, procedural errors, visual–spatial difficulties, metacognitive misconceptions, and reasoning-and-proof difficulties.

Among these categories, conceptual errors emerged as the most frequently reported obstacle. Students often relied on visual appearance when determining similarity rather than examining proportional relationships and angle correspondence (Nafiin, 2020; Nurmala, S., & Jupri, 2023; Solihah & Muhtadi, 2025). Similar patterns have been documented internationally, where learners tend to interpret geometric figures perceptually instead of relationally (Gutiérrez, A., & Jaime, 2021; Jones & Tzekaki, 2016; Stylianides et al., 2024).

The thematic synthesis further suggests that students' errors are not isolated events but form a developmental sequence. Visual–spatial difficulties frequently precede conceptual misunderstandings, which subsequently contribute to transformation errors, procedural mistakes, and difficulties in deductive reasoning. Recent studies have similarly reported that weaknesses in visual interpretation significantly influence students' ability to establish proportional relationships and construct mathematical arguments (Mosher, 2022; Sinclair & Bruce, 2015; Stylianides et al., 2024).

Consequently, student errors should be interpreted not merely as failures but as indicators of evolving mathematical understanding. This perspective aligns with contemporary mathematics education research that views misconceptions as productive stages in conceptual development and learning progression (Alonzo & von Aufschnaiter, 2018; Hiebert et al., 2018; Stylianides et al., 2024).

2. Interpretation Through Mathematical Learning Theories

a. Van Hiele's Theory of Geometric Thinking

The prevalence of conceptual and visual–spatial difficulties strongly supports Van Hiele's theory of geometric thinking

(Hiele, 1986). According to this framework, students progress through hierarchical levels of reasoning from visualization to rigor.

The reviewed studies indicate that many students remain at the visualization level, where geometric judgments are primarily based on appearance rather than on invariant geometric relationships. Students frequently assumed that figures were similar because they appeared visually alike, despite failing to verify proportional side relationships (Firdaus & Mutaqin, 2019; Solihah & Muhtadi, 2025).

Recent empirical investigations continue to demonstrate that insufficient progression through Van Hiele levels remains a major factor contributing to difficulties in geometry learning (Bermudez & Graysay, 2025; Gutiérrez, A., & Jaime, 2021; Stylianides et al., 2024). Furthermore, Duval's theory of semiotic representation suggests that geometric understanding requires coordination among visual, symbolic, and conceptual representations. When such coordination is absent, students tend to privilege visual perception over mathematical reasoning (Duval, 1998; Sinclair & Bruce, 2015).

The reasoning-and-proof difficulties identified in this review further indicate that many students have not successfully transitioned toward informal and formal deductive reasoning. Similar findings have been reported in recent studies showing that students frequently struggle to justify similarity relationships using accepted geometric theorems and logical arguments (Ani Daniyati et al., 2023; Bermudez & Graysay, 2025).

b. Piagetian Cognitive Development

From a Piagetian perspective, the persistent reliance on visual resemblance suggests that many learners continue to operate within forms of reasoning associated with the concrete operational stage (Inhelder, B., & Piaget, 1958; Piaget, 1970b).

Geometric similarity requires proportional and relational reasoning, which are characteristics of formal operational thinking. However, the reviewed studies consistently revealed that students focused on observable shape characteristics rather than abstract proportional relationships (Nafiin, 2020; Rohmah & Rosyidi, 2022).

Recent research confirms that proportional reasoning remains one of the most significant predictors of success in geometry and algebra learning (Stark Guss et al., 2022; Stylianides et al., 2024). Therefore, difficulties in learning similarity may partly reflect developmental readiness rather than solely instructional deficiencies.

These findings support recommendations advocating the use of manipulatives, dynamic geometry environments, and guided abstraction strategies that facilitate students' transition from concrete representations to formal mathematical reasoning (Battista, 2007; Hohenwarter et al., 20007; Sinclair & Bruce, 2015).

c. Polya's Problem-Solving Model

Errors observed in contextual Transformation and procedural errors can be understood through Polya's problem-solving framework (Polya et al., 1945). Many students experienced difficulties during the stages of understanding the problem and devising a plan, particularly when solving contextual similarity tasks.

Students frequently failed to identify corresponding quantities correctly before constructing proportional relationships, resulting in inaccurate mathematical models (Fadilah & Bernard, 2021; Rohmah & Rosyidi, 2022). Such findings suggest that students' difficulties originate not primarily from computation but from the interpretation and representation of problem situations.

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Recent studies in mathematical modelling similarly indicate that students often struggle to translate real-world contexts into appropriate mathematical structures (Mosher, 2022; Stylianides et al., 2024). Schoenfeld's metacognitive perspective further suggests that successful problem solving requires continuous monitoring and evaluation of solution strategies rather than the mechanical application of procedures (Schoenfeld, 1985).

d. Vygotsky's Sociocultural Perspective

The findings also support Vygotsky's sociocultural theory, particularly the concept of the Zone of Proximal Development (ZPD) (Vygotsky, 1978). Several studies revealed that students possessed partial conceptual understanding but encountered difficulties when applying concepts independently.

Teacher guidance, peer discussion, and collaborative reasoning emerged as important mechanisms for supporting conceptual development (Ditinjau et al., 2021; Murio, 2022). Recent studies have similarly demonstrated that structured scaffolding can significantly improve students' geometric reasoning and proportional understanding (Sinclair & Bruce, 2015; Stylianides et al., 2024). These findings highlight the importance of learning environments that promote mathematical communication, collaborative exploration, and gradual release of responsibility.

e. Newman's Error Analysis Framework

The synthesis based on Newman Error Analysis revealed that transformation and comprehension errors were the most frequently reported categories across the reviewed studies. Students often failed to identify relevant information, understand problem requirements, and construct appropriate mathematical representations before beginning calculations (Laila et al., 2025; Nafiin, 2020; Nurmala, S., & Jupri, 2023).

Recent research supports the view that many mathematical errors originate during representational and interpretative stages rather than computational stages (Audétat et al., 2017; Bapa et al., 2025). Consequently, Newman's framework provides a valuable diagnostic lens for identifying the specific stages at which students' reasoning begins to break down.

3. Integrative Theoretical Synthesis

Taken together, the five theoretical frameworks provide complementary explanations for the developmental pattern of errors identified in this review. Piaget explains students' tendency to rely on concrete visual reasoning. Van Hiele specifies the level of geometric thinking attained. Polya identifies where difficulties emerge during problem solving. Vygotsky explains how social interaction facilitates conceptual advancement. Newman identifies the specific stages at which errors occur.

Rather than functioning independently, these frameworks collectively suggest that students' errors represent developmental manifestations of mathematical learning. This interpretation aligns with recent perspectives emphasizing learning trajectories, conceptual growth, and productive use of misconceptions as resources for instruction (Alonzo & von Aufschnaiter, 2018; Bakker & Smit, 2018; Hiebert et al., 2018; Mosher, 2022).

4. Implications for the Development of a Hypothetical Learning Trajectory (HLT)

A major contribution of this study is the construction of a Hypothetical Learning Trajectory (HLT) for geometric similarity based on empirical evidence synthesized across the reviewed studies. The proposed HLT suggests a progression from visual recognition of geometric figures, identification of corresponding elements, development of proportional reasoning, application in contextual situations, and finally deductive justification and proof construction. This sequence reflects the developmental pattern observed in the literature and is consistent with contemporary HLT research in mathematics education (Irawan et al., 2025; Nogueira et al., 2021; Stark Guss et al., 2022).

The findings suggest that instruction should not begin with formal similarity theorems but should first strengthen visual and conceptual understanding before introducing increasingly abstract reasoning processes.

5. Pedagogical Implications

The findings imply that effective instruction on geometric similarity should:

a) Emphasize conceptual understanding before procedural fluency.
b) Integrate error-based learning activities to promote metacognitive awareness (Borasi, 1994; Goldberg, 2016).
c) Utilize dynamic geometry software such as GeoGebra to strengthen visual analytical connections (Hohenwarter et al., 20007; Sinclair & Bruce, 2015).
d) Incorporate collaborative scaffolding and mathematical discourse (Goos et al., 2002; Stylianides et al., 2024).
e) Introduce proof progressively through exploratory and semi-deductive activities before formal proof construction (Bermudez & Graysay, 2025; Sowder, 2007).

6. Research Gaps and Future Directions

Future research should focus on:

a) Longitudinal investigations of students' geometric reasoning development.
b) Experimental validation of the proposed HLT.
c) Technology-supported scaffolding interventions.
d) Relationships among metacognition, proportional reasoning, and misconceptions.
e) Development of AI-assisted diagnostic systems for geometry learning.

7. Concluding Remarks

This review demonstrates that learning geometric similarity is a complex developmental process involving conceptual, visual, procedural, metacognitive, and deductive dimensions. Students' difficulties emerge through interconnected stages, beginning with visual interpretation and extending to formal reasoning and proof.

By integrating insights from Van Hiele, Piaget, Polya, Vygotsky, and Newman, this study provides a comprehensive explanation of how students' difficulties develop and how instruction can respond to those difficulties. The resulting Hypothetical Learning Trajectory offers an empirically grounded framework for designing learning experiences that support students' progression from intuitive visual judgments toward formal geometric reasoning and proof.

CONCLUSION

This Systematic Literature Review provides a comprehensive synthesis of empirical evidence regarding students' difficulties in learning geometric similarity and their implications for the development of a Hypothetical Learning Trajectory (HLT). The findings reveal that students' difficulties are multidimensional and interconnected, encompassing conceptual errors, transformation errors, procedural errors, visual–spatial difficulties, metacognitive misconceptions, and reasoning-and-proof difficulties. These difficulties do not occur independently; rather, they form a developmental pattern in which weaknesses in visual interpretation and conceptual understanding often lead to subsequent difficulties in mathematical representation, problem solving, and deductive reasoning.

The synthesis further indicates that many students remain at the visualization and early analytical levels of geometric thinking, where judgments about similarity are primarily based on perceptual appearance rather than proportional reasoning and formal geometric relationships. Such findings suggest that students' misconceptions should not be viewed solely as indicators of failure but as manifestations of ongoing cognitive development that require instructional support aligned with learners' developmental readiness.

From a theoretical perspective, this study integrates insights from five major frameworks in mathematics education—Van Hiele's levels of geometric thinking, Piaget's cognitive development theory, Vygotsky's sociocultural perspective, Polya's problem-solving framework, and Newman's Error Analysis. Collectively, these perspectives provide a multidimensional explanation of how and why students experience difficulties in learning geometric similarity. The synthesis demonstrates that students' errors emerge from the interaction between developmental readiness, geometric reasoning levels, representational competence, problem-solving processes, and social mediation.

A major contribution of this study is the development of an evidence-informed Hypothetical Learning Trajectory (HLT) for geometric similarity derived from the systematic synthesis of empirical literature. The proposed trajectory suggests a progressive learning pathway consisting of: (1) visual recognition of geometric figures, (2) identification of corresponding elements, (3) development of proportional reasoning, (4) application of similarity in contextual situations, and (5) deductive justification and proof construction. This trajectory provides a conceptual framework for designing learning experiences that are responsive to students' learning difficulties and cognitive development.

The findings also generate several pedagogical implications. Effective instruction on geometric similarity should move beyond procedural teaching and place greater emphasis on conceptual understanding, error-based learning, reflective reasoning, collaborative scaffolding, and the use of dynamic visualization tools such as GeoGebra. Such approaches may facilitate students' transition from intuitive visual judgments toward formal deductive reasoning and deeper mathematical understanding.

Despite these contributions, several limitations should be acknowledged. First, the review was limited to studies published between 2015 and 2025 and included only articles written in English and Indonesian, which may have excluded relevant findings reported in other languages or earlier publications. Second, the proposed HLT was developed through secondary data synthesis and has not yet been empirically validated in classroom settings. Third, variations in research designs, participant characteristics, educational contexts, and reporting practices across the reviewed studies may influence the generalizability of the synthesized findings.

Therefore, future research should focus on empirically testing and refining the proposed HLT through classroom-based interventions, design experiments, and longitudinal studies across different educational levels and cultural contexts. Further investigations may also explore the integration of digital technologies, adaptive learning environments, and AI-assisted diagnostic systems to support students' geometric reasoning and conceptual development.

In conclusion, this study demonstrates that learning geometric similarity is a complex developmental process involving conceptual, representational, visual, procedural, metacognitive, and deductive dimensions. By synthesizing empirical evidence and established theoretical perspectives, this review contributes a theoretically grounded and empirically informed Hypothetical Learning Trajectory that can support future instructional design, research, and practice in mathematics education. Ultimately, students' errors should be understood not merely as obstacles to learning but as valuable sources of information for guiding meaningful conceptual growth and mathematical reasoning development.

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AUTHOR CONTRIBUTIONS

The first author was responsible for the conceptualization of the study, conducting the investigation, performing formal analysis, managing data curation, drafting the original manuscript, and preparing visual materials. The second and third authors contributed to methodological development, validation of the findings, critical review and editing of the manuscript, and overall supervision of the research process.

REFERENCES

Al-baqie, A. F. A., & Budiarto, M. T. (2022). *Kemampuan Membaca , Menulis , dan Membuktikan Siswa dalam Menyelesaikan Masalah Kesebangunan Dua Segitiga*. *5*(1), 110–126. https://doi.org/10.30762/f

Alonzo, A. C., & von Aufschnaiter, C. (2018). Moving Beyond Misconceptions: Learning Progressions as a Lens for Seeing Progress in Student Thinking. *The Physics Teacher*, *56*(7), 470–473. https://doi.org/10.1119/1.5055332

Ani Daniyati, Ismy Bulqis Saputri, Ricken Wijaya, Siti Aqila Septiyani, & Usep Setiawan. (2023). Konsep Dasar Media Pembelajaran. *Journal of Student Research*, *1*(1), 282–294. https://doi.org/10.55606/jsr.v1i1.993

Artigue, M. (2009). Didactical design in mathematics education. *ZDM Mathematics Education*, *41 (1-2)*, 13–23.

Audétat, M., Laurin, S., Dory, V., Charlin, B., Laurin, S., Dory, V., & Charlin, B. (2017). Diagnosis and management of clinical reasoning difficulties : Part I . Clinical reasoning supervision and educational diagnosis. *Medical Teacher*, *39*(8), 792–796. https://doi.org/10.1080/0142159X.2017.1331033

Bakker, A., & Smit, J. (2018). Using hypothetical learning trajectories in design research. In *Design Research in Education* (pp. 255–271). Routledge.

Bapa, A. T., Susanto, H., & Malang, U. N. (2025). *Symmetry | Pasundan Journal of Research in Mathematics Learning and Education*. *10*(2007), 31–43. https://doi.org/10.23969/symmetry.v10i1.21959

Battista, M. (2007). The development of geometric and spatial thinking. In *The Cambridge Handbook of Mathematics Education Research.*

Bermudez, H., & Graysay, D. (2025). Supporting Students in Exploring, Developing, and Formalizing Mathematical Arguments. *PRIMUS*, *35*(1), 102–133. https://doi.org/10.1080/10511970.2024.2445517

Borasi, R. (1994). Capitalizing on Errors as "Springboards for Inquiry": A Teaching Experiment. *Journal for Research in Mathematics Education*, *25*(2), 166–208. https://doi.org/10.5951/jresematheduc.25.2.0166

Clements, D. H., & Battista, M. T. (1989). *Geometry and Spatial Reasoning*. https://doi.org/10.1108/978-1-60752-874-620251022

Didi, S., Muhammad, P., & Endang, M. (2019). Hypothetical Learning Trajectory Of Students On Learning Geometry In Junior High School Grade 7Th. *Advances in Social Science, Education and Humanities Research*, *287*(Icesre 2018), 180–184. https://doi.org/10.2991/icesre-18.2019.38

Ditinjau, K., Siswa, D., Hakiim, S., & Hidayati, F. H. (2021). *Problematika Pembelajaran Daring Materi Kesebangunan dan*. *1*, 44–52.

Dreyfus, T. (2015). Constructing abstract mathematical knowledge in context. In S. J. Cho (Ed.), Selected Regular Lectures from the 12th International Congress on Mathematical Education(pp. 115-133). *Springer International Publishing.*, 115–133. https://doi.org/https://doi.org/10.1007/978-3-319-17187-6_7

Duval, R. (1998). Geometry from a cognitive point of view. *Educatinal Studies in Mathematics*, *31 (1-2)*, 1–25.

Fadilah, R., & Bernard, M. (2021). ANALISIS KESALAHAN SISWA DALAM MENYELESAIKAN MASALAH MATEMATIKA KONTEKSTUAL MATERI KEKONGRUENAN DAN KESEBANGUNAN. *Jurnal Pembelajaran Matematika Inovatif*, *4*(4), 817–826. https://doi.org/10.22460/jpmi.v4i4.81

7-826

Firdaus, A., & Mutaqin, A. (2019). *DESAIN DIDAKTIS MATEMATIS PROBLEM SOLVING PADA (MATHEMATICAL DIDACTIC DESIGN OF PROBLEM SOLVING ON*. *02*(01), 16–27.

Ginting, A. B., Aulia, D. U., Anindya, I. B., & Fauzi, M. A. (2025). ANALISIS KESALAHAN SISWA DALAM MENYELESAIKAN SOAL GEOMETRI MATERI KEKONGRUENAN DAN KESEBANGUNAN BERDASARKAN TEORI NEWMAN. *Jurnal Review Pendidikan Dan Pengajaran*, *8*(1), 3206–3214. https://doi.org/10.31004/jrpp.v8i1.38421

Goldberg, S. R. (2016). Examining Students ' Metacognitive Awareness Through Analysis of Student- generated Learning Responses. *ASEE Annual Conference & Exposition*, 16. https://doi.org/10.18260/p.26795

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition. *Educational Studies in Mathematics*, *49 (2)*, 195–223.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics*, *49*, 193–223.

Gravemeijer, K. & Cobb, P. (2006). *Design Research from a Learning Design Perspective". In J. Van den Akker, K. Gravemeijer, S. McKenney & N. Nieveen (Eds). Educational Design Research*.

Gravemeijer, K., & Doorman, M. (1999). Educational Studies in Mathematics,. *Educational Studies in Mathematics*, *39 (1-3)*, 111–129.

Gutiérrez, A., & Jaime, A. (2021). Revisiting Van Hiele's model in light of current research on geometry thinking. *International Journal of Science and Mathematics Education*, *19 (16)*, 1021–1040. https://doi.org/https://doi.org/10.1007/s10763-021-10194-6

Hamzah, E., T, A. Y., & Nursangaji, A. (2019). ANALISIS KESALAHAN KONSEP DALAM MENJAWAB SOAL-SOAL PADA MATERI KESEBANGUNAN MENGGUNAKAN CERTAINTY OF RESPONSE INDEX. *Jurnal Pendidikan Dan Pembelajaran Khatulistiwa (JPPK)*, *2003*, 1–9. https://doi.org/https://doi.org/10.26418/jppk.v8i11.37595

Hiebert, J., Morris, A. K., & Spitzer, S. M. (2018). Diagnosing Learning Goals: An Often-Overlooked Teaching Competency. In *Diagnostic Competence of Mathematics Teachers* (pp. 193–206). https://doi.org/10.1007/978-3-319-66327-2_10

Hiele, V. (1986). Structure and Insight: A Theory of Mathematics Education. In *Academic Press*.

Hohenwarter, M., Hohenwarter, J., & Kreis, Y. (20007). Teaching and calculus with free dynamic mathematics software GeoGebra GeoGebra and Open Source. *In 11th International Congress on Mathematical Education (ICME 11).*

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking: From childhood to adolescence.* https://doi.org/https://doi.org/10.1037/10034-000

Irawan, D., Marpaung, M. T. F., Dewi, I., & Surya, E. (2025). Paradigms of Mathematics Education Research: A Theoretical Review and Implications for Learning Practice. *International Journal Of Humanities Education and Social Sciences (IJHESS)*, *5*(1). https://doi.org/10.55227/ijhess.v5i1.1898

Jones, K., & Tzekaki, M. (2016). *4 . RESEARCH ON THE TEACHING AND LEARNING OF GEOMETRY.*

Laila, F. M., Helina Qatrunnada Nasution, Lisda Karolin Br Barus, & Kms Muhammad Amin Fauzi. (2025). Analisis Kesalahan Mahasiswa Dalam Menyelesaikan Soal Kesebangunan Dan Kekongruenan Di Jurusan Matematika Universitas Negeri

Medan. *Jurnal Pedagogy*.

Mariotti, M. A., & Fischbein, E. (1997). DEFINING IN CLASSROOM ACTIVITIES. *Educational Studies in Math.Ematics*, *34*, 219–248. file:///C:/Users/ASUS/Downloads/defESMcopia.pdf

Mosher, F. A. (2022). Learning Progressions. In *Learning Progressions*. https://doi.org/10.4324/9781138609877-REE115-1

Murio, N. (2022). *Nazom Murio: Mahasiswa FKIP Universitas Jambi Page 1*. 1–8.

Nafiin, M. W. (2020). ANALISIS KESALAHAN PESERTA DIDIK DALAM MENYELESAIKAN SOAL KESEBANGUNAN. *Jurnal Ilmiah Pendidikan Matematika*, *9*(2), 21–29. https://doi.org/https://doi.org/10.26740/mathedunesa.v7n2.p485-492

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks', in M. A. Clements, and J. Foyster (eds.),. *Research in Mathematics Education in Australia, 1977*, *1*, 239–258.

Nogueira, V. B., Teixeira, D. G., de Lima, I. A. C. N., Moreira, M. V. C., de Oliveira, B. S. C., Pedrosa, I. M. B., de Queiroz, J. W., &, & Jeronimo, S. M. B. (2021). Towards an inclusive digital literacy: An experimental intervention study in a rural area of Brazil. *Education and Information Technologies, 27 (2)*, 2807–2834.

Nurmala, S., & Jupri, A. (2023). Common errors and cognitive obstacles in learning similarity. *Journal on Mathematics Education*, *14 (2)*, 155–172. https://doi.org/https://doi.org/10.22342/jme.14.2.17791.155-172

OECD. (2024). *PISA 2022 Results Volume III : Creative Minds , Creative Schools*. *III*.

Osibodu, O., & Sunghwan Byun. (2021). A Participatory Turn in Mathematics Education Research: Possibilities and Tensions. *National Council of Teachers of Mathematics*, *54*(3). https://doi.org/https://doi.org/10.5951/jresematheduc-2021-0147

Osta, I. (2014). Mathematics Curriculum Evaluation. In *Encyclopedia of Mathematics Education* (pp. 417–423). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_168

Ouzzani, M. (2016). Rayyan — a web and mobile app for systematic reviews. *Systematic Reviews*, *2016*, 1–10. https://doi.org/10.1186/s13643-016-0384-4

Page, M. J., Mckenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-wilson, E., Mcdonald, S., … Moher, D. (2021). *The PRISMA 2020 statement : an updated guideline for reporting systematic reviews Systematic reviews and Meta-Analyses*. https://doi.org/10.1136/bmj.n71

Patton, M. (2015). *Qualitative Research and Evaluation Methods. 4th Edition, Sage, Thousand Oaks.*

Piaget. (1970a). *Cognitive Developmental Standpoint*.

Piaget, J. (1970b). *Science of Education and the Psychology of the Child*.

Polya, G., To, H., It, S., Principle, F., & Principle, S. (1945). *Polya ' s Problem Solving Techniques*. 1–4.

Riyan, H., Mohd, S. M. R., Normala, I., & Noor, M. A. M. (2023). Systematic Review of Spatial Thinking in Mathematics Education. *Educational Studies: Conference Series*, *3*(1), 47–60.

Rohmah, D. I., & Rosyidi, A. H. (2022). *MATHE dunesa*. *11*(3).

Saldaña, Matthew B.П, M., Johnny, & Huberman, A. M. (2014). *Qualitative Data Analysis A Methods Sourcebook Edition 3*. SAGE Publications. https://www.metodos.work/wp-content/uploads/2024/01/Qualitative-Data-Analysis.pdf

Schoenfeld, A. H. (1985). Mathematical problem solving. *New York: Academic Press.*

Silmi, U., & L, D. A. M. (2022). Systematic Literature Review:Teori Van Hiele dalam Meningkatkan Kemampuan Berpikir Geometris Siswa Sekolah Dasar. *PEDADIDAKTIKA: Jurnal Ilmiah Pendidikan Guru Sekolah Dasar*, *9*(2), 327–338. https://doi.org/10.17509/pedadidaktika.v9i2.53166

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, *26*(2), 114–145.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM Mathematics Education*, *47*(3), 319–329. https://doi.org/10.1007/s11858-015-0693-4

Snyder, H. (2019). Literature review as a research methodology: An overview and guidelines. *Journal of Business Research*, *104*, 333–339. https://doi.org/10.1016/j.jbusres.2019.07.039

Solihah, S., & Muhtadi, D. (2025). *Kesulitan konseptual dan prosedural siswa smp dalam materi kesebangunan Pendahuluan*. *04*(01), 123–140.

Sowder, L. (2007). Second Handbook of Research on Mathematics Teaching and Learning,. In *Nantional Council of Teachers of Mathematics* (Vol. 6179, Issue 619, pp. 1–60).

Stark Guss, S., Clements, D., & Sarama, J. (2022). High-quality early math: Learning and teaching with trajectories and technologies. In *Handbook of Research on Innovative Approaches to Early Childhood Development and School Readiness*. https://doi.org/https://doi.org/10.4018/978-1-7998-8649-5.ch015

Stylianides, G. J., Stylianides, A. J., & Moutsios, A. (2024). Proof and proving in school and university mathematics education research : a systematic review. *ZDM – Mathematics Education*, *56*(1), 47–59. https://doi.org/10.1007/s11858-023-01518-y

Sulastri, R., Meutia, H., Johar, R., & Ikhsan, M. (2017). PROSES PEMBELAJARAN MATEMATIKA. *SEMINAR NASIONAL KEMARITIMAN ACEH*, *1*, 285–288.

Tall, D. (2004). Introducing Three Worlds of Mathematics. *For the Learning of Mathematics*, *23*, 29–33.

Tall, D. (2011). Crystalline Concepts in Long-Term Mathematical Invention and Discovery. *For the Learning of Mathematics*, *31 (1)*, 3–8. https://doi.org/https://eric.ed.gov/?id=EJ921149

Usiskin, Z. (1982). Van Hiele levels and achievement in secondary school geometry. *University of Chicago*.

Vygotsky, L. S. (1978). Mind in Society: The Development of Higher Psychological Processes. *Cambridge, MA: Harvard University Press.*